Introduction

We solved some problems of the GEOMATECH competition on the workshop.

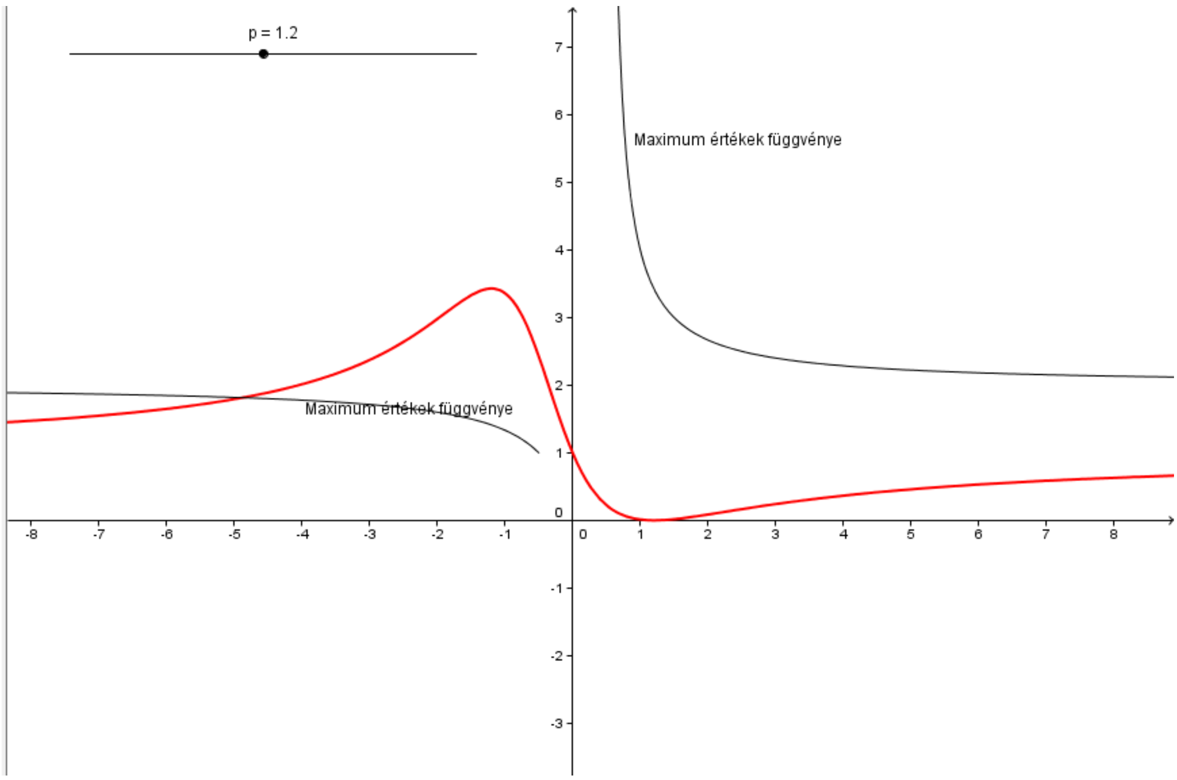
The children sent in the most creative solutions, when they got totally free, constitutive tasks. The 11th and 12th grade students had to advertise the GEOMATECH project in the first round. We got many interesting solutions. I have to highlight Talentum group’s solution because they used a lot of geometrical objects, transformations and made an imaginative, colourful advertisement using GeoGebra dinamic possibilies. I had an idea in the same round, when I saw the presentation of the Geohb group's school: to construct the school's building from squares with lists and rotations. I show two solutions on the worksheet.

The task of the 9th and 10th grade students in the second round show how the dynamic possibilities of GeoGebra can be used for discussion solving a geometry problem.

In the second round the task of the 11th and 12th grade students was graphing a parametric function. It is an interesting problem because we get completely different graphs (in terms of continuity, boundedness, limit) depending on the parameters. We asked the traditional solution on paper too. The drawing in GeoGebra helped them to obtain a conjecture before the exact solution as to what graphs will be created for certain parameter values.

Round 2

11th and 12th class students



We really need the slider of GeoGebra to answer the following questions for parametric function.

Plot the following function in GeoGebra!

$$f(x) = \frac{(x-p)^2}{x^2+x+p^2}$$

Let the domain of this funcion be the widest possible subset of the real numbers, where p is a real parameter, p$\in$ [-5; 8].

We can get significantly different functions for/with different p values. Substantially different means now that with different value of p the function may differ according to continuity, the number of break points, boundedness, monotonicity, and parity terms.

Characterize the obtained substantially different functions!

Then determine for which values of parameter p is the function limited? How much is the maximum and the minimum of the function in these cases?

How much is the smallest possible value of the maximum and with which parameter value is it realized?

Let's formulate conjectures based on the function representation, then let's prove them!

1. Solution -- on paper -- traditional method – exact solution

2 break points, if $|p| < \frac{1}{2}$

1break point, if $p = \pm\frac{1}{2}$, $f(x) = \frac{\left(x-\frac{1}{2}\right)^2}{\left(x+\frac{1}{2}\right)^2}$,

$p = -\frac{1}{2}$ $f(x) = 1$, the function is bounded

1 break point if $p = 0, f(x) = \frac{x^2}{x^2+x} = \frac{x}{x+1}$

There isn't breakpoint if $|p| > \frac{1}{2}$, $\lim\limits_{x\to\pm\infty} f(x) = 1$, the function here bounded.

$$f'(x) = \frac{(x-p)\cdot(x+p)\cdot(2p+1)}{(x^2+x+p^2)^2}$$

Extreme value $x = \pm p$

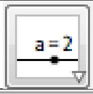
Min: $f(p) = 0$,

Max: $f(-p) = \frac{4p}{2p-1} = 2 + \frac{2}{2p-1}$

if $p \in \left]\frac{1}{2}, +\infty\right[$ then $f(-p) \in \left]+\infty, 2\right[$

if $p \in \left]-\infty, -\frac{1}{2}\right[$, then $f(-p) \in \left]2,1\right[$

2. Solutin – with GG, only illustration, conjecture

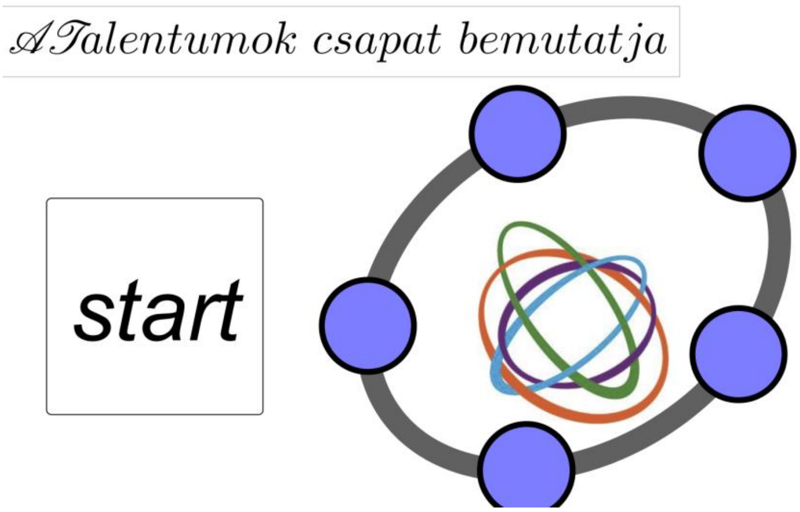
	Tool	Definition	Comment
1		min: -5, max: 8 Increment: 0.01	
2	Parancssor:	f(x)=(x-p)^2/(x^2+x+p^2)	
3	Parancssor:	g(x)=4x/(2x-1)	function of maximums
4	Parancssor:	Function[g(x), -100, -0.5] Function[g(x), 0.5, 100]	Function[<Function>, <Start x-Value>, <End x-Value>]
5	Parancssor:	derivative[f], derivative[f,2]	1. and 2. derivative

Talentum team – GEOMATECH advertisement Competition task 1st round, 11th 12th class

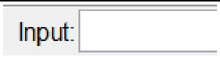
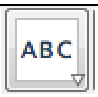
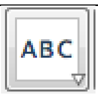
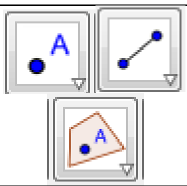
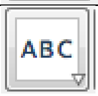
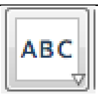
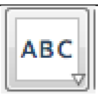
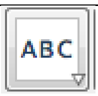
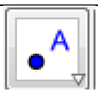
Related file: talentumok_11_12.ggb

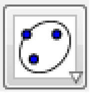
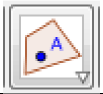
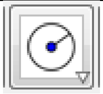
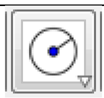
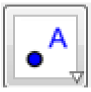
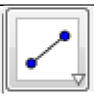
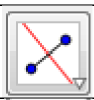
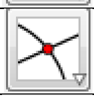
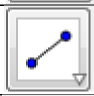
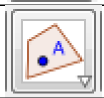
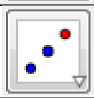
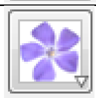
Imagine working in a advertising design studio. During design use as many dynamic opportunities of GG as you can (e.g. colours, slider, check box, button, input box and there should also be connections among the shapes)

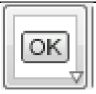
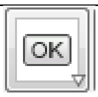
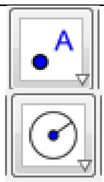
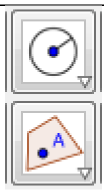
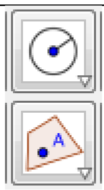
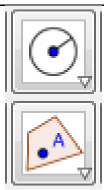
Design a commercial for GEOMATECH project with GG. Make this commercial as interesting, unusual and creative as possible.

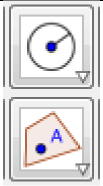
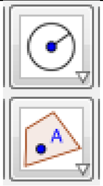
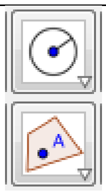
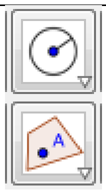
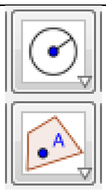


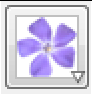
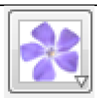
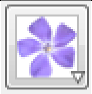
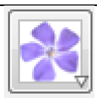
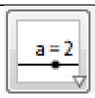
First we all shapes draw, all picture, text paste, then show object out, and we adjust condition to show object only at the end of work.

	Tool	Definition	Comment
1		Polygon[Corner[1], Corner[2], Corner[3], Corner[4]] colour:black opacity 100 Advanced: Condition to show object: $0 < n < 40 \vee 43 < n < 50$	Name:poly1
2		Name: text1 large, light blue Advanced: Condition to show object: $n \stackrel{?}{=} 1$	
3		Name: text2 extra large, gold Advanced: Condition to show object: $n \stackrel{?}{=} 2$	
4		A B Segment[A, B] C=Point[j], Animation On	j=Segment[A,B] Point on j
5		Name: text3 medium, gold Position:starting point C Advanced: Condition to show object: $2 < n < 26$	After every sentence follows a blank line
6		Name: text4 very large, serif, gold Advanced: Condition to show object: $n \stackrel{?}{=} 0$	
7		Name: text5 very large, serif, gold Advanced: Condition to show object: $n \stackrel{?}{=} 50$	
8		Name: text6 large, sans-serif, gold Advanced: Condition to show object: $43 < n < 50$	After every sentences follow a blank line
9		D E F	

10		Ellipse with foci D, E passing through F Colour 5/8 grey, line thickness 13 Advanced: Condition to show object: n $\stackrel{?}{=}$ 0	c
11	 or 	G=Point[c], H=Point[c], I=Point[c], J=Point[c], K=Point[c] Animation On	Animation on after 13
12		Circle with center G, radius 10 Circle with center H, radius 10 Circle with center I, radius 10 Circle with center J, radius 10 Circle with center K, radius 10 Colour light blue, line thickness 1, Opacity 100 Advanced: Condition to show object: n $\stackrel{?}{=}$ 0	
13		Circle with center G, radius 10 Circle with center H, radius 10 Circle with center I, radius 10 Circle with center J, radius 10 Circle with center K, radius 10 Colour black, opacity 0, line thickness 13 Advanced: Condition to show object: n $\stackrel{?}{=}$ 0	
14	 	A_2 A_1 s_1=Segment[A_2,A_1]	
15		Perpendicular bisector of s_1	g
16		s_1, g	M
17		Segment[M, A_1]	h
18		Point on h	S_2
19		S_2 reflected in M	S_1
20		Name:pic1 Position: Corner1: S_1 Corner2: S_2 Advanced: Condition to show	

		object: $n \stackrel{?}{=} 0$	
21		Name: button1 Caption: Start extra large width: 240 px, heigh: 240 px Advanced: Condition to show object: $n \stackrel{?}{=} 0$ Scripting: On Click: StartAnimation[n]	
22		Name: button2 Caption: Menu extra large width: 240 px, heigh: 130 px Advanced: Condition to show object: $n \stackrel{?}{=} 50$ Scripting: On Click: n=0	
23		Sun_1 Sun =Circle with center Sun _1 and Radius 8 Colour yellow opacity 100 Advanced: Condition to show object: 26<n<39	Sun The condition applies only to circle (to point not)
24		Merkur_2=Circle with center Sun _1 and Radius 10 Colour white opacity 0 Merkur_1=Point on Merkur_2 Merkur= Circle with center Merkur_1 and Radius 1 Colour grey, opacity 100 Advanced: Condition to show object: 26<n<39	Mercur The condition applies only to 2 circles (to point not) The planets are in different shades of yellows and blues
25		Venus_2=Circle with center Sun _1 and Radius 12 Colour white opacity 0 Venus_1=Point on Venus_2 Venus= Circle with center Venus_1 and Radius 1.5 Colour yellow opacity 100 Advanced: Condition to show object: 26<n<39	Venus The condition applies only to 2 circles (to point not)
26		Earth_2=Circle with center Sun _1 and Radius 16 Colour white Opacity 0 Earth _1=Point on Earth _2 Earth = Circle with center Earth _1 and Radius 2 Colour blue	Earth The condition applies only to 2 circles (to point not)

		Opacity 100 Advanced: Condition to show object: 26<n<39	
27		Mars_2=Circle with center Sun _1 and Radius 21 Colour white, opacity 0 Mars _1=Point on Mars _2 Mars = Circle with center Mars _1 and Radius 2.4 Colour maroon, opacity 100 Advanced: Condition to show object: 26<n<39	Mars The condition applies only to 2 circles (to point not)
28		Jupiter _2=Circle with center Sun _1 and Radius 30 Colour white, opacity 0 Jupiter _1=Point on Jupiter _2 Jupiter = Circle with center Jupiter _1 and Radius 6 Colour yellow, opacity 100 Advanced: Condition to show object: 26<n<39	Jupiter The condition applies only to 2 circles (to point not)
29		Saturn _2=Circle with center Sun _1 and Radius 42 Colour white opacity 0 Saturn _1=Point on Saturn _2 Saturn = Circle with center Saturn _1 and Radius 5 Colour yellow opacity 100 Advanced: Condition to show object: 26<n<39	Saturn The condition applies only to 2 circles (to point not)
30		Uranus _2=Circle with center Sun _1 and Radius 50 Colour white opacity 0 Uranus _1=Point on Uranus _2 Uranus = Circle with center Uranus _1 and Radius 3 Colour blue opacity 100 Advanced: Condition to show object: 26<n<39	Uranus The condition applies only to 2 circles (to point not)
31		Neptune _2=Circle with center Sun _1 and Radius 60 Colour white opacity 0 Neptune _1=Point on Neptune _2 Neptune = Circle with center Neptune _1 and Radius 2.8 Colour blue opacity 100 Advanced: Condition to show object: 26<n<39	Neptune The condition applies only to 2 circles (to point not)

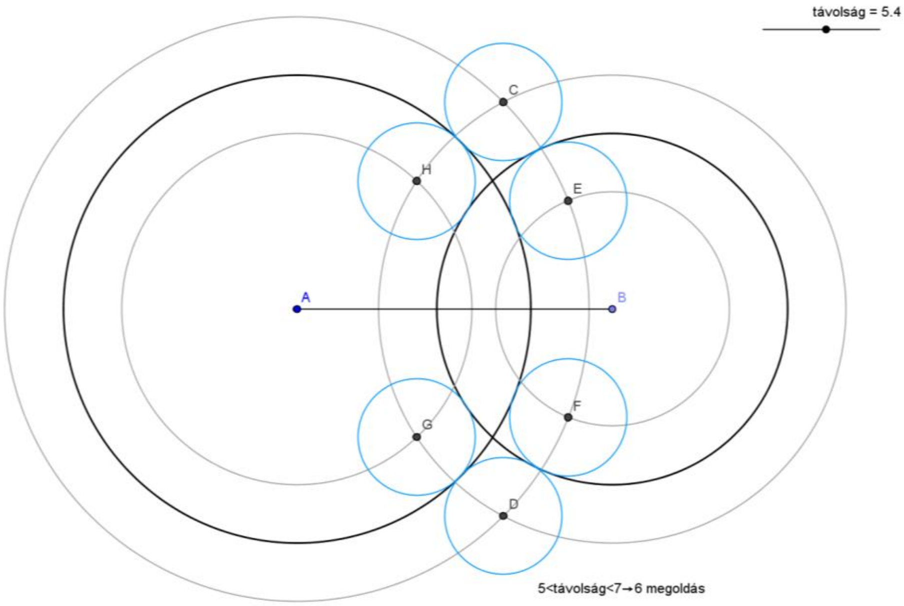
32		Name:pic2 Advanced: Condition to show object: $n \stackrel{?}{=} 40$	
33		Name:pic3 Advanced: Condition to show object: $n \stackrel{?}{=} 41$	
34		Name:pic4 Advanced: Condition to show object: $n \stackrel{?}{=} 42$	
35		Name:pic5 Advanced: Condition to show object: $n \stackrel{?}{=} 43$	
36		Name: n Min:0, Max: 50, Increment:1, speed 0.1, repeat: Increasing (once) Scripting: On Update If[30<n<38,ZoomIn[2,Earth_1]] If[n == 39, ZoomIn[1/128, Earth_1]]	

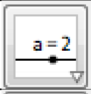
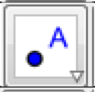
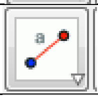
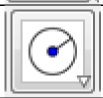
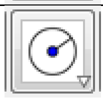
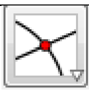
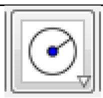
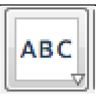
Round 2

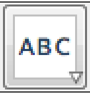
9th and 10th class student

Discussion

A circle with a radius of 4 units and another one with a radius of 3 units are given. Construct a circle or circles with a radius of 1 unit that touches both given circles! How does the number of solutions change when we change the distance of the two centers of the given circles? Analyse the task in detail according to the number of the solutions!



	Tool	Definition	Comment
1		Min: 0, Max:10, Increment:0.1	Rename -- distance
2		A point	
3		A point, distance	B point
4		Circle with centre A, radius 4, Circle with centre B, radius 3,	c,d
5		Circle with centre A radius 3 and 5, Circle with centre B radius 2 and 4 colour blue	e, f, g, h
6		Intersection point of e and f Intersection point of e and g, etc.	C, D, …points (when all 8 point visible)
7		Circle with centre C radius 1, Circle with centre D radius 1, etc.	
8		Text: If distance < 1, or distance > 9 → no solution Advanced –Condition to show object: (distance < 1) ∨ (distance > 9)	

9		and so on the other texts…	

Geohb team's school construction with list

Competition task, 1st round, Grade 7, 8

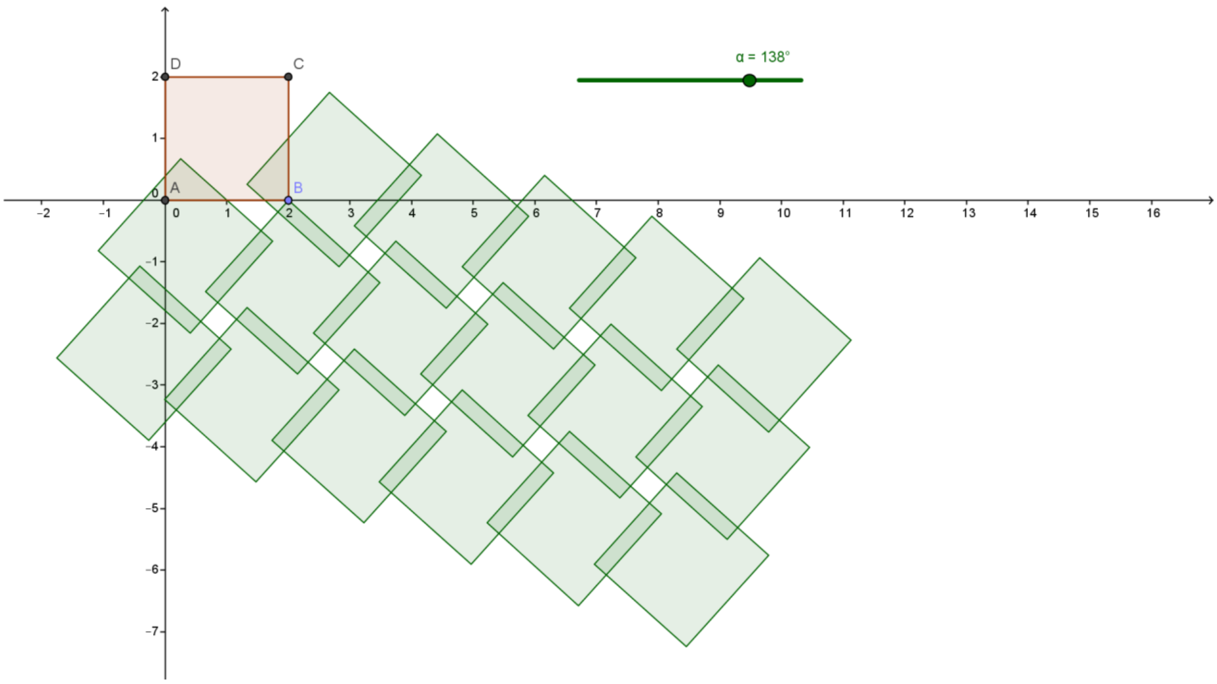
In the first month the task of upper primary school students is introduction. We would like to learn more about you, so we are asking you to introduce yourselves in a GeoGebra drawing. During drawing use as many geometrical shapes, geometrical transformations (reflection, translation, rotation) as you can. On the drawing apply colours and animations.

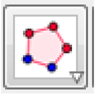
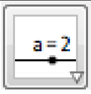
We would like to know where you study. Draw your school or classroom in GeoGebra. Use moving parts on the drawing.

Related files: geohb_7_8_2.ggb, forgatás listával1.ggb, forgatás listával2.ggb

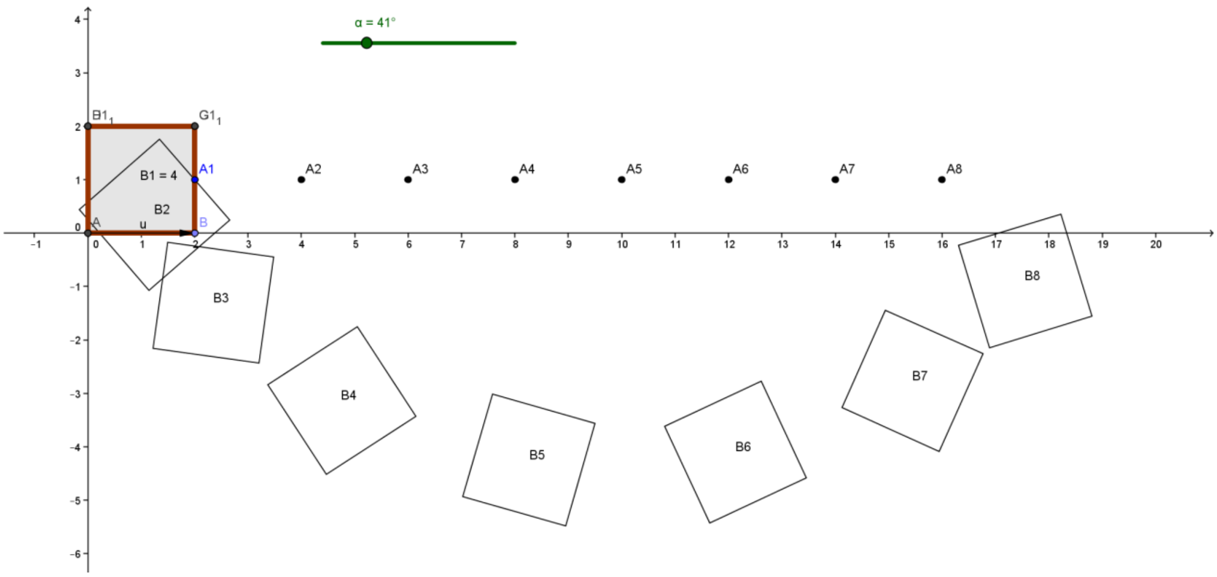
Note: The children did not use list in the solution, they rotated the squares one by one

First solution: We rotate the same square around different points

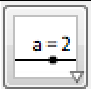
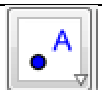
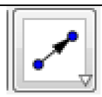


	Tool	Definition	Comment
1		A=(0,0), B=(2,0)	poly1
2		Angle, min: 0°, max: 180°, increment: 1°	
3	Input:	Sequence[Rotate[poly1, α, (i, 1)], i, 1, 6]	Rotation poly1 square with α angle, around (i,1) centres. 1st square series
4	Input:	Sequence[Rotate[poly1, α, (i, 0)], i, 1, 6]	Rotation of poly1 square with α angle, around (i,0) centres. 2nd square series
5	Input:	Sequence[Rotate[poly1, α, (i, -1)], i, 1, 6]	Rotation of poly1 square with α angle, around (i,-1) centres. 3rd square series

Second solution: We get each square with the rotation of it's preceded square. The centers of rotations are different. We write the rotation centers and the squares in the spreadsheet, too.



Open view menu, spreadsheet

	Tool	Definition	Comment
1		Angle, min: 0°, max: 180°, increment: 1°	
2		A=(0,0), B=(2,0)	definition of 2 vertices of the first square
3	Spreadsheet A1 cell	(2,1)	definition of the first rotation centre
4		u=[A, B] vector	
5	Spreadsheet A2 cell	Translation[A1,u]	definition of the second rotation centre
6		A2 cell copy, pulling by right lower corner(as in Excel)	definition further rotation centres
7	Spreadsheet B1 cell	Polygon[A, B,4]	Definition first square. If we don't see the square in the graphics view, right button in the cell, show object
8	Spreadsheet B2 cell	Rotate[B1, α, A1]	Rotation of square (which is in the cell B1) with angle α, around point A1
9		copy cell B2, pulling the right lower corner	Thus, we get the following: Rotation of square (which is in cell B2) with angle α, around A2 point Rotation of square (which is in cell B3) with angle α, around point A3, etc.